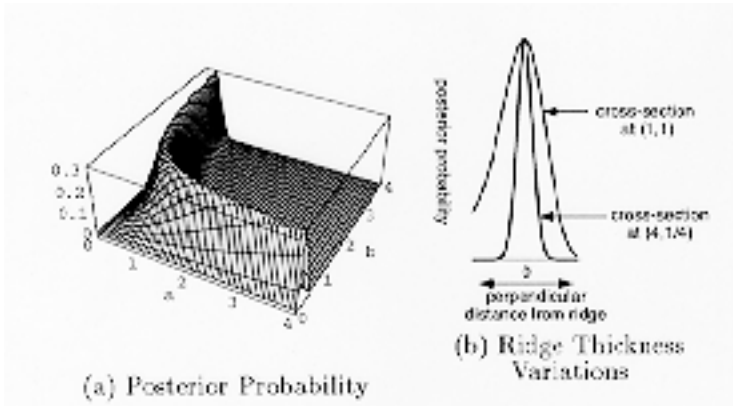
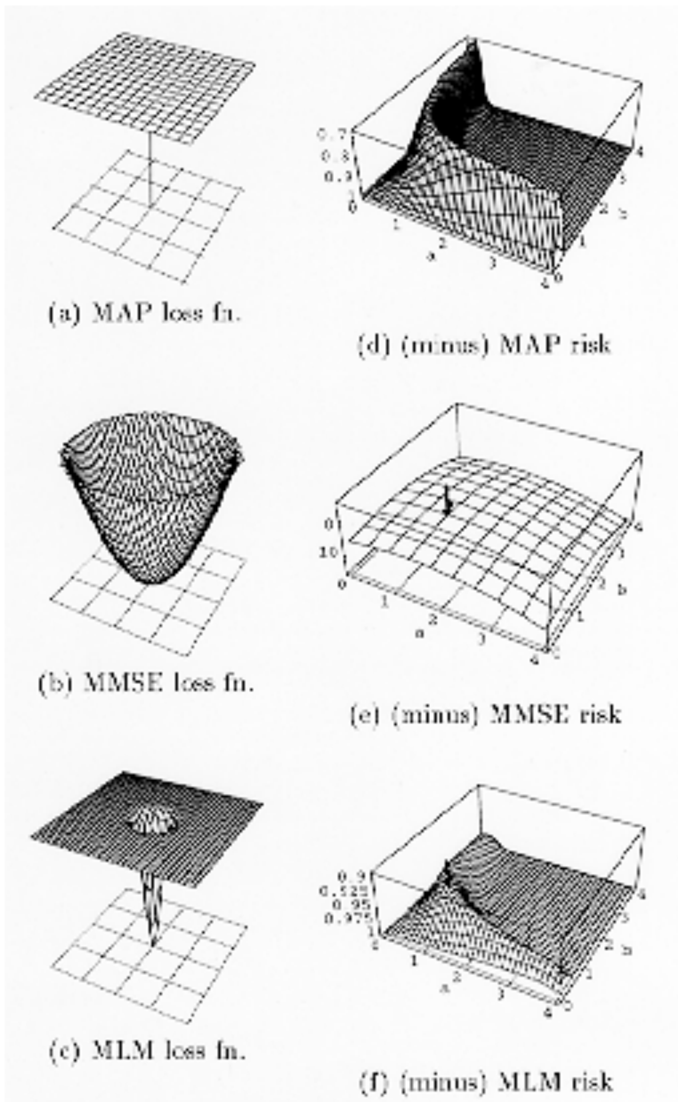
Bayesian decision theory, the maximum local mass method, and color constancy



Bayesian analysis of the problem a b = 1.

Assuming uniform prior probabilities over the graphed region, (a) shows the posterior probability for gaussian observation noise of variance \$0.18\$. The noise broadens the geometric solution into a hyperbola--shaped ridge of maximum probability. (b) Note the different thickness of the ridge; some parts have more local probability mass than others, even though the entire ridge has a constant maximum height.



Three loss functions, and the expected loss

Left column: Three loss functions. Plots show penalty for guessing parameter values offset from the actual value, taken to be the plot center. (a) Minus delta function loss, assumed in MAP estimation. Only {\em precisely} the correct answer matters. (b) Squared error loss (a parabola), used in MMSE estimation. Very wrong guesses can carry inordinate influence. (c) Minus local mass loss function. Nearly correct answers are rewarded while all others carry nearly equal penalty. Right column: Corresponding expected loss, or Bayes risk, for the y = a b problem. Note: loss increases vertically, to show extrema. (d) Expected loss for MAP estimator is minus the posterior probability. There is no unique point of minimum loss. (e) The minimum mean squared error estimate, (1.3, 1.3) (arrow) does not lie along the ridge of solutions to a b = 1. (f) The minus local mass loss favors the point (1.0, 1.0) (arrow), where the ridge of high probability is widest. There is the most probability mass in that local neighborhood.

References

D. H. Brainard and W. T. Freeman, *Bayesian Color Constancy* , Journal of the Optical Society of America, A, 14(7), pp. 1393-1411, July, 1997. pdf file.

W. T. Freeman and D. H. Brainard, *Bayesian decision theory, the maximum local mass estimate, and color constancy*, Fifth Intl. Conference on Computer Vision, IEEE Computer Society, Cambridge, MA, U.S.A, June, 1995, pp. 210 - 217. TR94-23.

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